Algoritmi i Strukture podataka

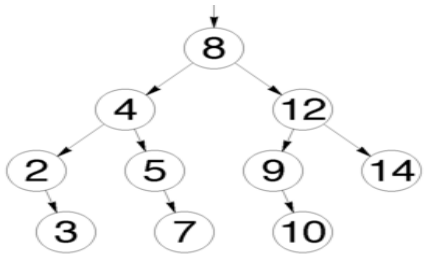
- ***Zadaci iz stabala-***

**** Svi zadaci su testirani****

```
** Klasa binarnog stabla
	package Branko-Stabla;
	public class BinarnoStablo {
		public CvorStabla koren;

		public BinarnoStablo() {
		koren = null;
	}
}
** Metoda koja broji elemente u stablu
	public int prebrojCvorove(CvorStabla cvor) {
		if (cvor == null) return 0;
		return 1+prebrojCvorove(cvor.levo) + prebrojCvorove(cvor.desno);
	}

** Metoda koja racuna zbir svih elemenata stabla
	public int zbirCvorova(CvorStabla cvor) {
		if (cvor == null)
			return 0;
		return  cvor.podatak+  zbirCvorova(cvor.levo)+zbirCvorova(cvor.desno);

	}
```

**Prefiksni prolaz kroz stablo se zasniva na prolazu kroz stablo tako da se prvo poseti koren, pa sva leva deca, pa sva desna deca

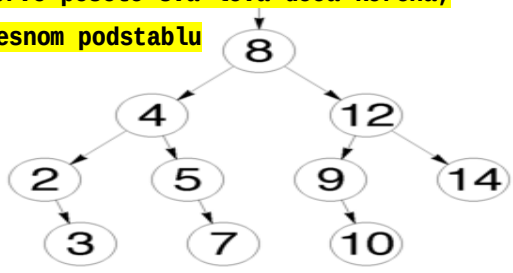
```
	public void prefiksniProlaz(CvorStabla cvor) {
		if (cvor == null)
			return;
		System.out.println(cvor.podatak);
		prefiksniProlaz(cvor.levo);
		prefiksniProlaz(cvor.desno);
	}
```



8, 4, 2, 3, 5, 7, 12, 9, 10, 14

** Infiksni prolaz kroz stablo je prolaz gde se prvo posete sva leva deca korena, pa sam koren, pa na kraju deca koja se nalaze u desnom podstablu

```
public void infiksniProlaz(CvorStabla cvor) {
		if (cvor == null)
			return;
		infiksniProlaz(cvor.levo);
		System.out.println(cvor.podatak);
		infiksniProlaz(cvor.desno);
```

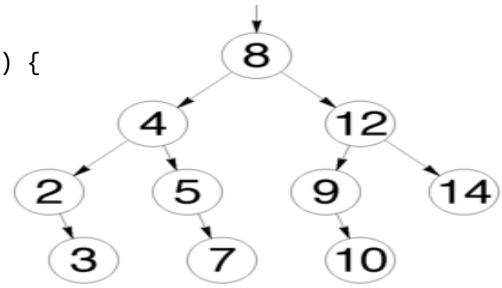


2, 3, 4, 5, 7, 8, 9, 10, 12, 14

```
	}
```

**** Postfiksni prolaz prvo ispisuje levu decu podstabla, potom desnu pa na kraju ispise koren stabla**

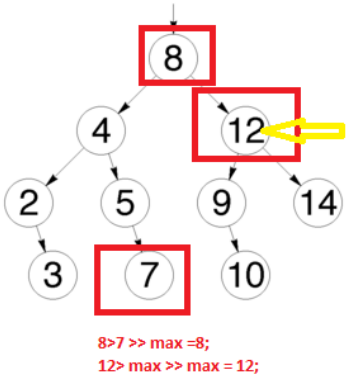
```
	public void postfiksniProlaz(CvorStabla cvor) {
		if (cvor == null)
			return;
		postfiksniProlaz(cvor.levo);
		postfiksniProlaz(cvor.desno);
		System.out.println(cvor.podatak);
	}
```



3, 2, 5, 7, 4, 10, 9, 14, 12, 8

Metoda koja nalazi i vraca vrednost maksimalnog elementa u stablu

```
	public int maxElement(CvorStabla cvor) {
		if (cvor == null)
			return Integer.MIN_VALUE;
		int max = cvor.podatak;
		int l = maxElement(cvor.levo);
		int d = maxElement(cvor.desno);

		if (max < l)
			max = l;
		if (max < d)
			max = d;
		return max;
	}
```



Metoda koja vraca najveci cvor u stablu

```
	public CvorStabla maxCvor(CvorStabla cvor) {
		if (cvor == null)
			return null;
		CvorStabla max = cvor;
		CvorStabla maxl = maxCvor(cvor.levo);
		CvorStabla maxd = maxCvor(cvor.desno);

		if (maxl != null)
			if (max.podatak < maxl.podatak)
				max = maxl;
		if (maxd != null)
			if (max.podatak < maxd.podatak)
				max = maxd;
```

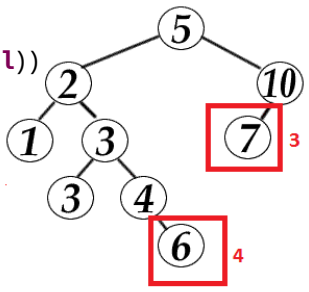
```
            return max;
      }
```

**Metoda koja pronalazi i vraca vrednost minimalnog elementa u stablu. Uslov za izlazak iz rekurzije je da ako je cvor stable null, da povratna vrednost bude Integer.*MAX_VALUE*. Pretpostavimo da je minimalni elemenat koren. Nakon toga nadjemo najmanji elemenat u levom podstabu, pa nakon toga u desnom podstablu, nakon cega sledi uporedjivanje korena na nadjenim minimumima. Vrednost najmanjeg od njih bice povratna vrednost metode.

```
      public int minElement(CvorStabla cvor) {
            if (cvor == null)
                  return Integer.MAX_VALUE;
            int min = cvor.podatak;
            int l = minElement(cvor.levo);
            int d = minElement(cvor.desno);

            if (min > l)
                  min = l;
            if (min > d)
                  min = d;
            return min;
      }
```

**Metoda koja racuna visinu stable. Visina je jednaka zbiru nivoa na kome se nalazi koren i dubini na kojoj se nalazi najdublji list

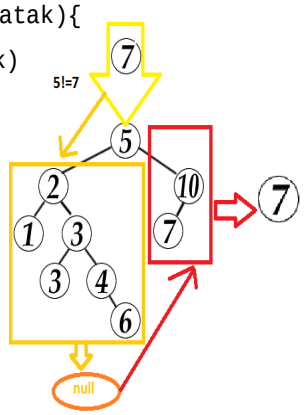
```
      public int visina(CvorStabla cvor) {
        if (cvor==null||(cvor.levo == null && cvor.desno == null))
            return 0;
      return 1 + max(visina(cvor.levo), visina(cvor.desno));
      }
      public int max(int a, int b) {
            if (a>b) {
                  return a;
            }
            return b; }
```



**Metoda koja pronalazi i vraca cvor koji sadrzi trazeni podatak. Najpre sledi provera da li je stablo prazno i da li je koren stable zapravo trazeni cvor. Pocetni uslov je zapravo i uslova za izlazak iz rekurzije. Nakon toga pokusamo pronaci elemenat u levom podstablu, i ako ga nadjemo vratimo tu vrednost. Ako ga

nismo pronasli u levom trazice se u desnom podstablu. Ako elemenat nit u nije pronadjen u skladu sa pocetnim uslovom metoda ce vratiti null.

```
public CvorStabla pronadji(CvorStabla tekuci, int podatak){
        if (tekuci == null || tekuci.podatak == podatak)
                return tekuci;
        CvorStabla l = pronadji(tekuci.levo, podatak);
                if (l != null)
                return l;
        return pronadji(tekuci.desno, podatak);
}
```



**** Metoda koja pronalazi i vraca trazeni cvor**

```
public CvorStabla pronadjiCvor(CvorStabla tekuci, CvorStabla P) {
        if (tekuci == null || tekuci==P)
                return tekuci;
        CvorStabla l = pronadjiCvor(tekuci.levo, P);
        if (l != null)
                return l;
        return pronadjiCvor(tekuci.desno, P);
}
```

**** Metoda koja vraca broj svih cvorova koji su veci od potomaka. Koren je veci od svojih potomaka ako je veci od levog i desnog deteta.**

```
public int prebroj (CvorStabla koren) {
if (koren == null || (koren.levo==null &&koren.desno==null))
                return 0;
if (koren.podatak>koren.levo.podatak && koren.podatak >koren.desno.podatak)
        return 1 + prebroj (koren.levo) + prebroj (koren.desno);
return prebroj(koren.levo)+prebroj(koren.desno);
}
```

**** Metoda koja vraca visinu na kome se nalazi trazeni cvor**

```
public int nivoDatogCvora (CvorStabla koren, CvorStabla p){
        if (koren==null) {
                return 0;
        }

        if(pronadjiCvor(koren.levo, p)!=null)
                return 1+nivoDatogCvora(koren.levo, p);

        if(pronadjiCvor(koren.desno, p)!=null)
                return 1+nivoDatogCvora(koren.desno, p);
```

```
		return 0;

	}
```

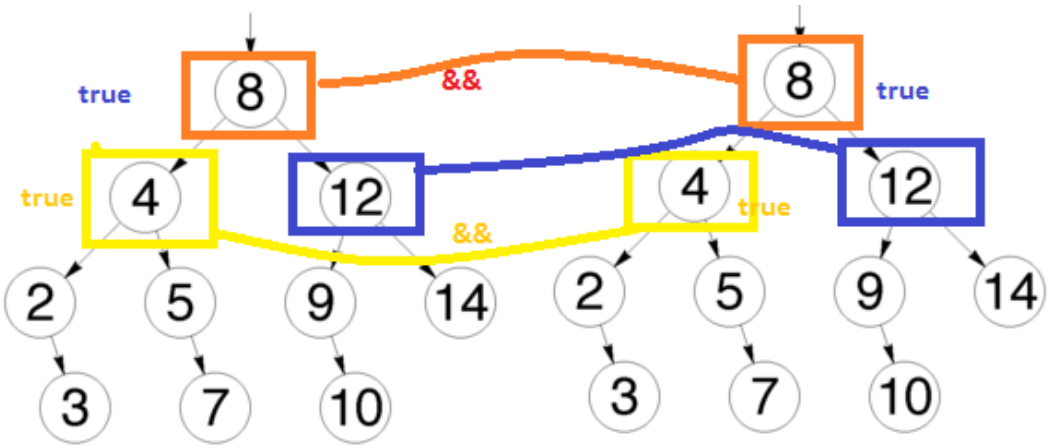
**Metoda vraca list koji se nalazi na najvecoj dubini.

```
	public CvorStabla najdubljiCvor(CvorStabla koren){
		if (koren==null || koren.levo==null && koren.desno==null) {
			return koren;
		}
		if (visina(koren.levo)>visina(koren.desno)) {
			return najdubljiCvor(koren.levo);
		}
		return najdubljiCvor(koren.desno);
	}
	public int visina(CvorStabla cvor) {
		if (cvor == null || (cvor.levo == null && cvor.desno == null))
			return 0;


		return 1 + max(visina(cvor.levo), visina(cvor.desno));
	}
	public int max(int a, int b) {
		if (a>b) {
			return a;
		}
		return b;
	}
```

**Metoda vraca najplici list

```
	public CvorStabla najpliciCvor(CvorStabla koren){

		if (koren==null|| (koren.levo==null && koren.desno==null)) {
			return null;
		}
		if (prviList(koren.levo)<prviList(koren.desno)) {
			return najpliciCvor(koren.levo);
		}
		return najpliciCvor(koren.desno);
	}

	public int prviList (CvorStabla koren){

		if (koren==null|| (koren.levo==null&&koren.desno==null)) {
			return 0;
		}
		return 1+ Math.min(prviList(koren.levo), prviList(koren.desno));
	}
```

Metoda proverava da li su dva stable identicna




```
	public boolean identicna(CvorStabla koren1, CvorStabla koren2){
		if (koren1==null && koren2==null ) {
			return true;
		}
		if (koren1!=null && koren2!=null) {
			if(koren1.podatak== koren2.podatak)
				return identicna (koren1.levo, koren2.levo) &&
					identicna (koren1.desno, koren2.desno);
		}
		return false;
	}
```

**** Metoda proverava jesu li dva stable slicna u ogledalu**

```
	public boolean jesuLiSlicna (CvorStabla koren1, CvorStabla koren2){
		if (koren1==null && koren2==null ) {
			return true;
		}
		if (koren1!=null && koren2!=null) {
			if(koren1.podatak== koren2.podatak)
				return	jesuLiSlicna(koren1.levo,	koren2.desno)&&
jesuLiSlicna(koren1.desno, koren2.levo);
		}
	return false;
}
```

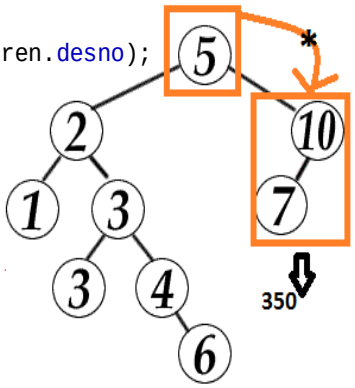
**** Broj cvorova koji imaju vecu vrednost od zadane**

```
	public int preboj (CvorStabla koren,int podatak){
		if (koren==null) {
```

```
                return 0;
            }
        if (koren.podatak>podatak)
            return 1+prebroj(koren.levo,podatak)+prebroj(koren.desno,podatak);

            return prebroj(koren.levo, podatak)+prebroj(koren.desno, podatak);
}
```

**Metoda racuna proizvod elemenata desnog podstabla

```
        public int proizvod (CvorStabla koren){
            if(koren==null)
                return 1;
        return koren.podatak*proizvod(koren.levo)*proizvod(koren.desno);
        }
        public int proizvodDesnoPodstabla (CvorStabla koren){
            if(koren==null|| koren.desno==null)
                return 0;
            return koren.podatak*proizvod(koren.desno);
}
```

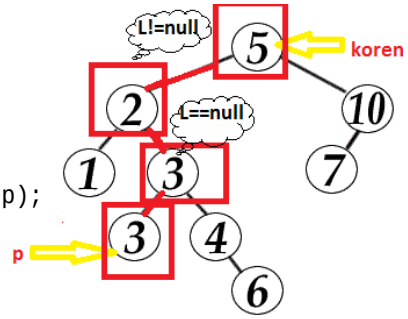


** Metoda iterativno ispisuje vrednosti od korena do zadatog cvora

```
        public void stampajOdKorenaDoZadatogCvora (CvorStabla koren, CvorStabla p){
            if(koren == null || koren==p)
                return;
            System.out.println(koren.podatak);
            while (koren!=p){
                if(pronadjiCvor(koren.levo, p)!=null){
                    System.out.println(koren.levo.podatak);
                    koren= koren.levo;
                    }else{
                        System.out.println(koren.desno.podatak);
                        koren=koren.desno;
                    }
                }
        }
```

Metoda rekurzivno stampa vrednosti u cvorovima od korena do datog cvora

```
        public void stampajOdKorenaDoCvoraRekurzivno(CvorStabla koren,CvorStabla
p){
        if(koren==null)
```




```
                return;
        System.out.println(koren.podatak);
        if (pronadjiCvor(koren.levo, p)!=null) {
           stampajOdKorenaDoCvoraRekurzivno(koren.levo, p);
                }

        if (pronadjiCvor(koren.desno, p)!=null) {
           stampajOdKorenaDoCvoraRekurzivno(koren.desno, p);
                }
}
```

**** Metoda ispisuje putanju od korena do cvora koji ima maksimalnu vrednost**

```
        public void stampajOdKorenaDoMaxCvora(){
                if(koren==null)
                        return;
                CvorStabla max = maxCvor(koren);
                stampajOdKorenaDoCvoraRekurzivno(koren, max);
                }
        public void stampajOdKorenaDoCvoraRekurzivno (CvorStabla koren, CvorStabla
p){
                if(koren==null)
                        return;
                System.out.println(koren.podatak);
                if (pronadjiCvor(koren.levo, p)!=null) {
                        stampajOdKorenaDoCvoraRekurzivno(koren.levo, p);
                        }

                if (pronadjiCvor(koren.desno, p)!=null) {
                        stampajOdKorenaDoCvoraRekurzivno(koren.desno, p);
                        }
                }
        public CvorStabla maxCvor(CvorStabla cvor) {
                if (cvor == null)
                        return null;
                CvorStabla max = cvor;
                CvorStabla maxl = maxCvor(cvor.levo);
                CvorStabla maxd = maxCvor(cvor.desno);
                if (maxl != null)
                        if (max.podatak < maxl.podatak)
                                max = maxl;
                if (maxd != null)
```

```
                if (max.podatak < maxd.podatak)
                        max = maxd;
        return max;
    }
```

**** Metoda racuna zbir vrednosti u cvorovima koji se nalaze na putanji od korena do zadatog cvora**

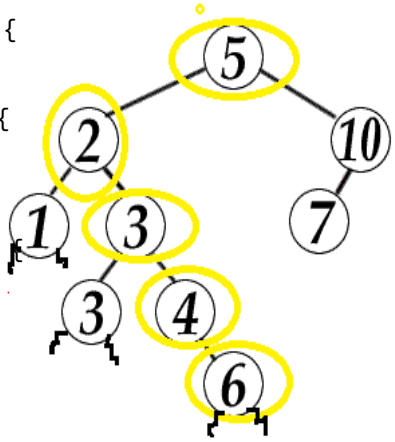
```
    public int zbirPutanja (CvorStabla pocetni, CvorStabla krajni) {
        if(pocetni==null)
            return 0;

        if (pronadjiCvor(pocetni.levo, krajnji)!=null) {
            return pocetni.podatak + zbirPutanja(pocetni.levo, krajnji);
        }
        if (pronadjiCvor(pocetni.desno, krajnji)!=null) {
            return pocetni.podatak + zbirPutanja(pocetni.desno, krajnji);
        }
        return krajnji.podatak;
    }
```

****Metoda stampa putanju izmedju dva zadana cvora u Avl stablu**

```
    public void stampajPutanjuAVLIzmedjuDvaCvora (CvorStabla koren, CvorStabla
    datiCvor) throws Exception{
        if(koren==null)
            throw new Exception("Greska");
        while(koren.podatak>datiCvor.podatak){
            System.out.println(koren.podatak);
            if(koren.podatak<datiCvor.podatak){
                koren=koren.desno;
                }
            else koren=koren.levo;
            }
        System.out.println(datiCvor.podatak);
    }
```

**** Metoda ispisuje putanju koja ima najvecu sumu od korena do lista**

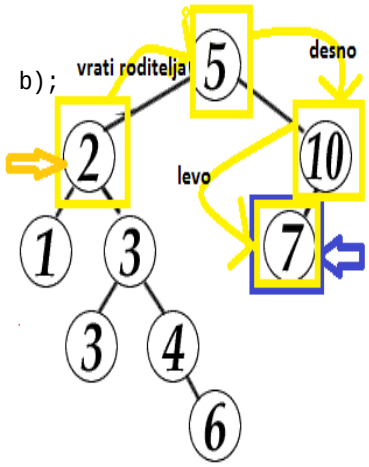
```
    public void ispisiputanjuSMaxSDoLista (CvorStabla k){
        System.out.println(k.podatak);
        if (k==null|| (k.levo==null&&k.desno==null)) {
            return;
        }
        if (zbirCvorova(k.levo)>zbirCvorova(k.desno)) {
            ispisiputanjuSMaxSDoLista(k.levo);
        }
        else
```



```
                ispisiputanjuSMaxSDoLista(k.desno);
        }

        public int zbirCvorova(CvorStabla cvor) {
                if (cvor == null)
                        return 0;
                return  cvor.podatak+  zbirCvorova(cvor.levo)+zbirCvorova(cvor.desno);

        }
```

****Metoda ispisuje putanju izmedju dva cvora u binarnom stablu**

```
        public void ispisiPutanju (CvorStabla k, CvorStabla a, CvorStabla b){
                if(k==null)
                        return;
                System.out.println(a.podatak);
                if (pronadjiCvor(a, b)==null) {
                  ispisiPutanju(k, vratiRoditelja(k, a), b);
                }
                if(pronadjiCvor(a.levo, b)!=null){
                   ispisiPutanju(k, a.levo, b);
                }
                if (pronadjiCvor(a.desno,b)!=null) {
                        ispisiPutanju(k, a.desno, b);
                }
           }
        }
```




```
        public CvorStabla pronadjiCvor(CvorStabla tekuci, CvorStabla P) {
                if (tekuci == null || tekuci==P)
                        return tekuci;
                CvorStabla l = pronadjiCvor(tekuci.levo, P);
                if (l != null)
                        return l;
                CvorStabla d = pronadjiCvor(tekuci.desno, P);
                if (d != null)
                        return d;
                return null;
                }
```

**** Metoda koja proverava da li je stablo BST**

```
        boolean daLiJeBst(CvorStabla koren) {
                if (koren == null)
```

```
                return true;
        if (koren.podatak > maxElement(koren.levo)
                        && koren.podatak < minElement(koren.desno)) {

                return daLiJeBst(koren.levo) && daLiJeBst(koren.desno);
        }
    return false; }

    public int maxElement(CvorStabla cvor) {
        if (cvor == null)
                return Integer.MIN_VALUE;
        int max = cvor.podatak;
        int l = maxElement(cvor.levo);
        int d = maxElement(cvor.desno);
        if (max < l)
                max = l;
        if (max < d)
                max = d;
    return max;
    }
```

**** Metoda proverava da li je stablo AVL**

```
    public boolean daLiJeAvl(CvorStabla cvor) {
        if (cvor == null)
                return true;

        if (daLiJeBst(cvor)) {
                int rv = visina(cvor.levo) - visina(cvor.desno);

                if (rv > -2 && rv < 2)
                        return daLiJeAvl(cvor.levo) && daLiJeAvl(cvor.desno);
        }
        return false;
    }
    public boolean daLiJeBst(CvorStabla koren) {
        if (koren == null)
                return true;
        if (koren.podatak > maxElement(koren.levo)
                        && koren.podatak < minElement(koren.desno)) {

                return daLiJeBst(koren.levo) && daLiJeBst(koren.desno);
        }

        return false;
    }

    public int maxElement(CvorStabla cvor) {
        if (cvor == null)
                return Integer.MIN_VALUE;
        int max = cvor.podatak;
```

```
		int l = maxElement(cvor.levo);
		int d = maxElement(cvor.desno);
		if (max < l)
			max = l;
		if (max < d)
			max = d;
		return max; }
```

Metoda koja vraca najveci cvor na putanji izmedju korena i cvora na koji je dat pokazivac

```
	public CvorStabla  vratiNajveciNaPutanji (CvorStabla k, CvorStabla b){

		if(k == null)
			return null;
		CvorStabla max = b;

		while (k!=b){
			if(k.podatak>max.podatak)
				max = k;
			if (pronadjiCvor(k.levo, b)!=null)
				k=k.levo;
			else
				k=k.desno;
		}

		return max;
	}
```

Metoda koja rekurzivno vraca najveci cvor na putanji izmedju korena i cvora na koji je dat pokazivac

```
	public CvorStabla vratiRekurzivno (CvorStabla k, CvorStabla b){
		if(k==null|| maxCvor(k)==k)
			return k;
		CvorStabla max=k;
		if(pronadjiCvor(k.levo, b)!=null){
			max = vratiRekurzivno (k.levo, b);
		}
		if(pronadjiCvor(k.desno, b)!=null){
			max = vratiRekurzivno (k.desno, b);
		}
		return max;
	}
	public CvorStabla pronadjiCvor(CvorStabla tekuci, CvorStabla P) {
		if (tekuci == null || tekuci==P)
			return tekuci;

		CvorStabla l = pronadjiCvor(tekuci.levo, P);
		if (l != null)
			return l;

		return pronadjiCvor (tekuci.desno, P);
		}
```

Dati su pokazivaci na koren i dva cvora u stablu. Metoda vraca zbir zajednickih predaha

```
	public int zbirPredaha (CvorStabla k, CvorStabla a, CvorStabla b){
		if(k==null)
			return 0;
		if (pronadjiCvor(k, a)!=null&& pronadjiCvor(k, b)!=null) {
			return k.podatak + zbirPredaha(k.levo,
a,b)+zbirPredaha(k.desno, a, b);
		}
		return 0;
	}
	public CvorStabla pronadjiCvor(CvorStabla tekuci, CvorStabla P) {
		if (tekuci == null || tekuci==P)
			return tekuci;

		CvorStabla l = pronadjiCvor(tekuci.levo, P);
		if (l != null)
			return l;

		return pronadjiCvor (tekuci.desno, P);
		}
```

B stabla

Ubacivanje:

– (1) Ako ima mesta u listu, uneti kljuc u list uz eventualna pomeranja

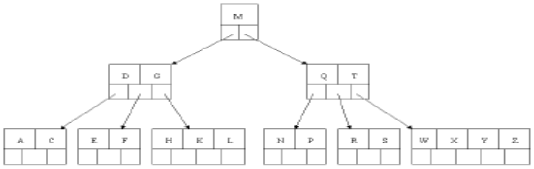
– (2) Ako je list pun, treba ga “pocepati”:
• (2.1) oko polovine kljuceva ostaje na starom listu
• (2.2) oko polovine kljuceva ide na novi list desno od postojeceg
• (2.3) srednji kljuc se “penje” u roditeljski cvor, pomeraju se kljucevi i pokazivac
– (3) Ako u roditeljskom (unutrašnjem) cvoru nema
mesta, i on se “cepa”:
• (3.1) oko polovine kljuceva ostaje na starom cvoru
• (3.2) oko polovine kljuceva ide u novi cvor desno od postojeceg
• (3.3.) srednji kljuc se “penje” u roditeljski cvor, uz pomeranje kljuceva i pokazivaca (rekurzivno korak 3)
– (4) Ako se “pocepa” koreni cvor, srednji kljuc ide u
novi koreni cvor; visina stabla se povecava za 1

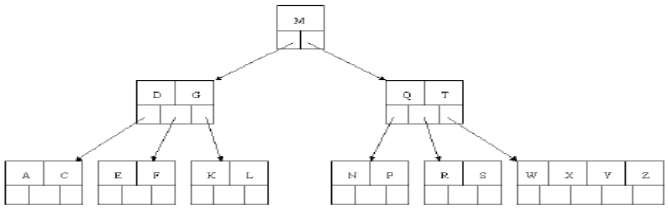
Izbacivanje:

• (1) Ako se briše kljuc iz lista onda
– (1.1) Ako ostaje dovoljno kljuceva, vrši se samo pomeranje
– (1.2) Ako ne ostaje dovoljno kljuceva onda
• (1.2.1) Ako susedni brat (cvor) ima “višak” kljuceva, najmanji kljuc iz desnog (odnosno najveci iz levog brata) se “penje” u cvor-prethodnik, a kljuc iz prethodnika se “spušta” u cvor iz kojeg smo izbrisali kljuc
• (1.2.2) Ako nijedan susedni brat nema “višak” kljuceva, cvor iz kojeg smo izbrisali kljuc spaja se sa levim ili desnim bratom, uz dodavanje “razdvojnog” kljuca iz cvora-prethodnika. Ako cvor-prethodnik nema dovoljno kljuceva, vrši se rotacija preko njegovog cvora-prethodnika, ili spajanje sa bratom

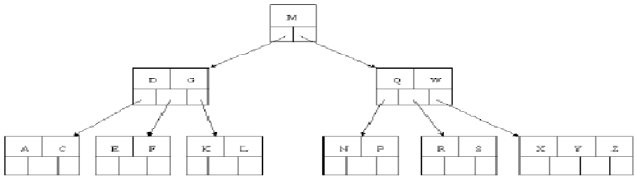
• (2) Ako se briše kljuc iz unutrašnjeg cvora, onda se umesto njega “penje” najmanji kljuc iz njegovog sledbenika; kljuc koji se penje briše se iz cvora u kome je bio – pravilo se primenjuje rekurzivno i
svodi na brisanje iz lista

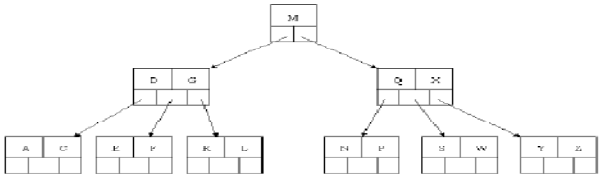
1.1 brise se H



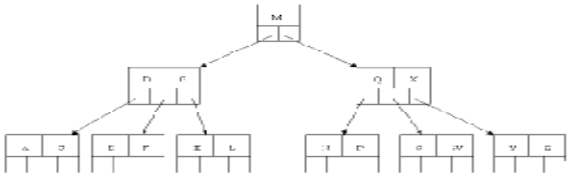


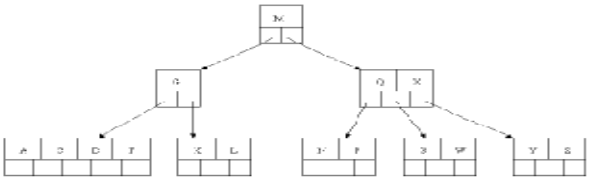
1.2.1 brise se R

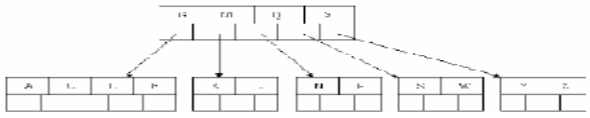




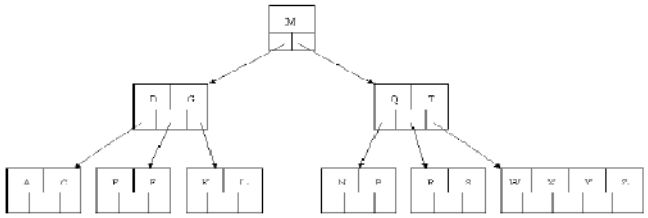
1.2.2 brise se E

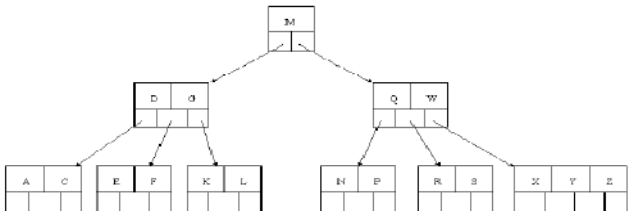




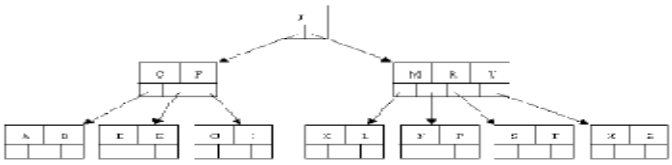


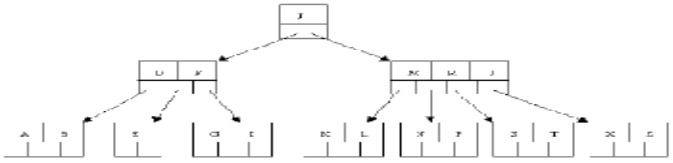
2. brise se T



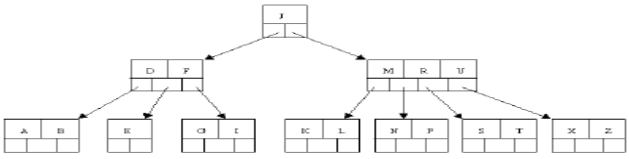


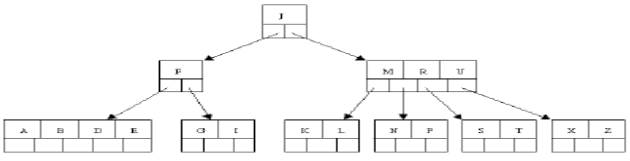
- Izbrisati ključ C – korak 1 (pravilo 2):



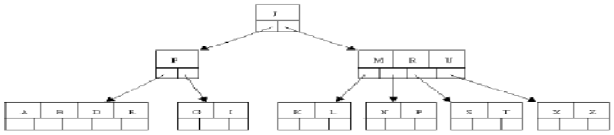


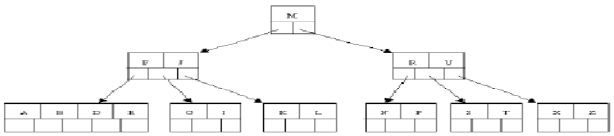
- Korak 2 (pravilo 1.2.2)





- Korak 3 (pravilo 1.2.2 – nast.). Čvor sa ključevima K, L promenio je prethodnika – i braću!



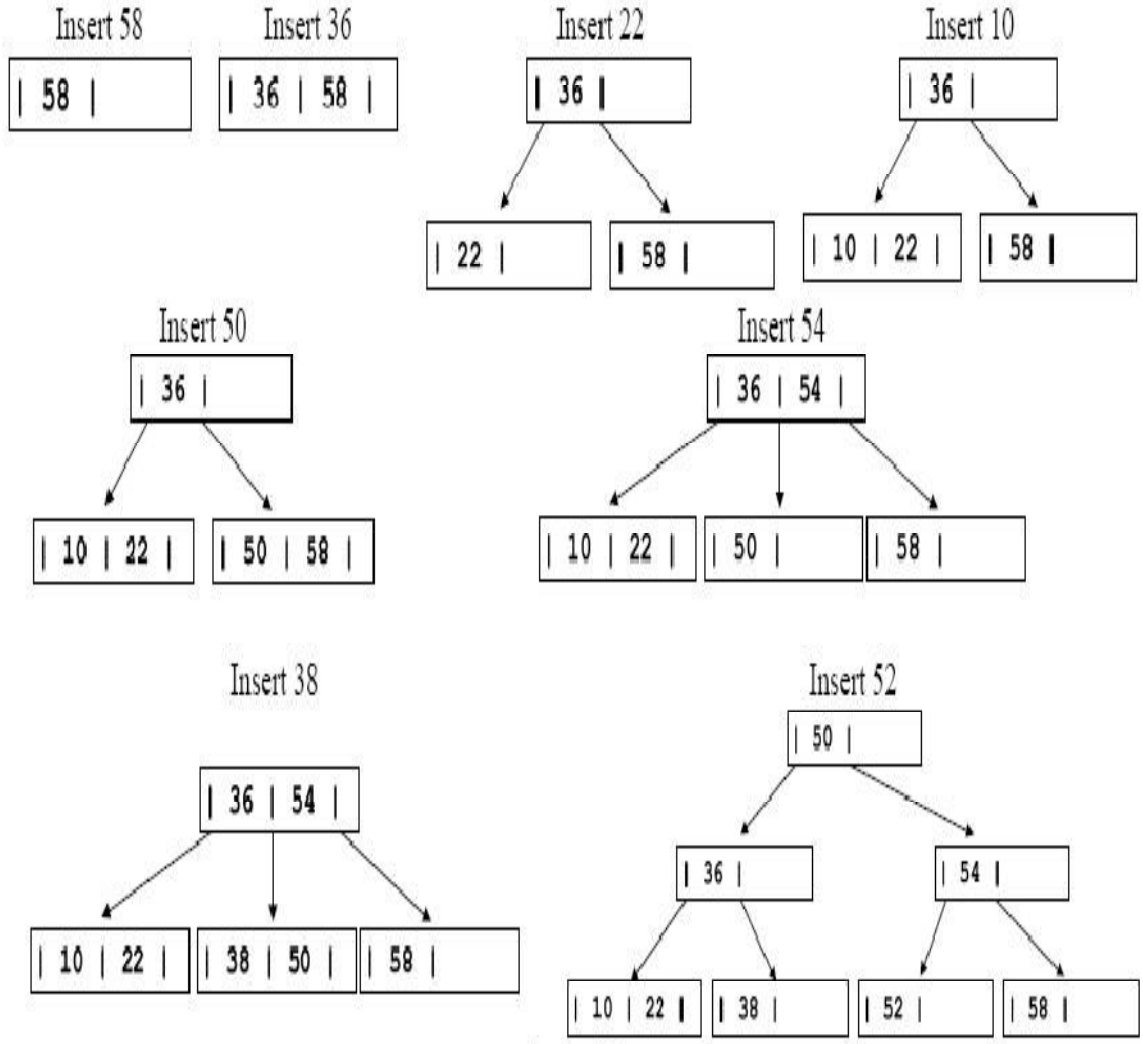


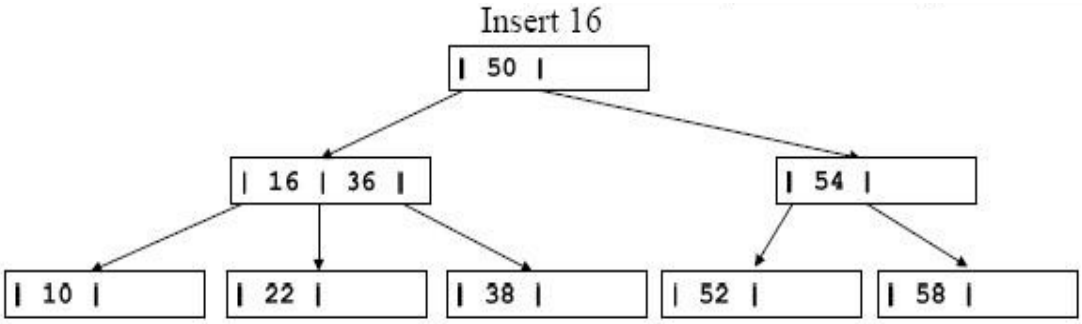
U inicijalno prazno B-stablo reda 3 umeću se redom ključevi 58, 36, 22, 10, 50, 54, 38, 52, 16, 40, 42, 12, 6, 27, 21 a zatim se redom brišu ključevi 50, 42, 38, 40, 52. Nacrtati izgled stabla nakon svake od navedenih izmena.

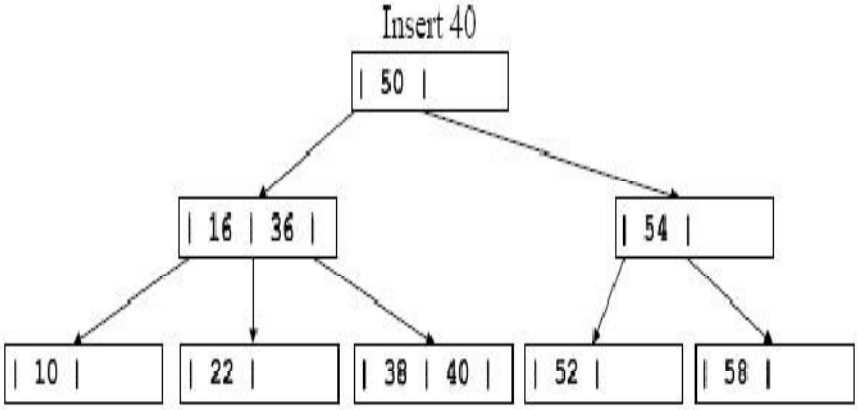
m=3

• svi čvorovi osim korena i listova imaju najmanje ⌈3/2⌉ = 2 podstabla

• listovi imaju najmanje ⌈3/2⌉ - 1 = 1 ključeva




Insert 16
50
16
36
54
10
22
38
52
58


Insert 40
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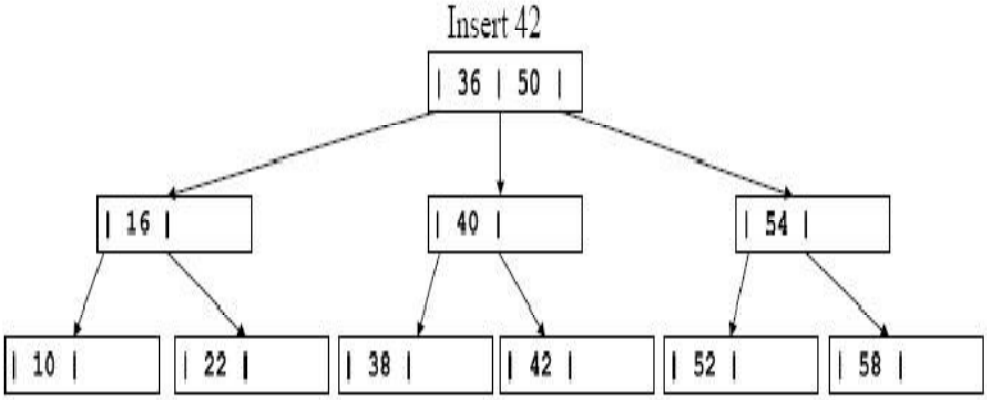
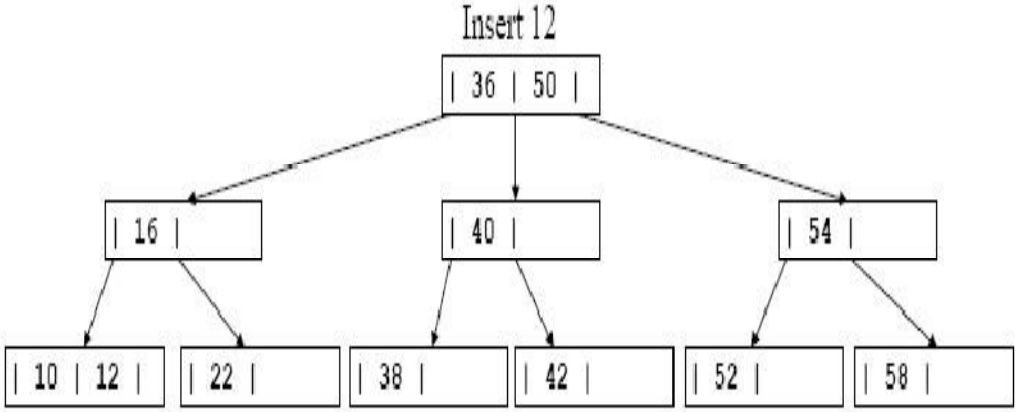
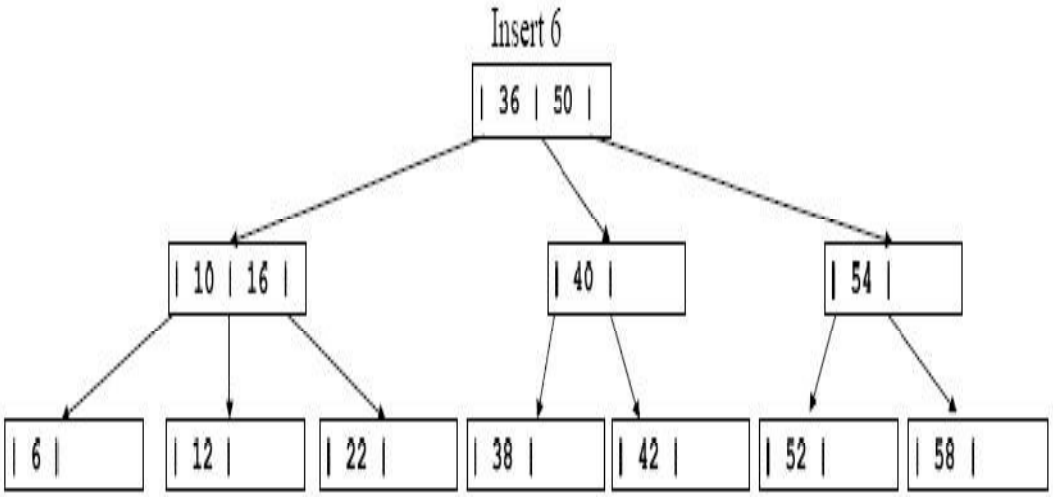
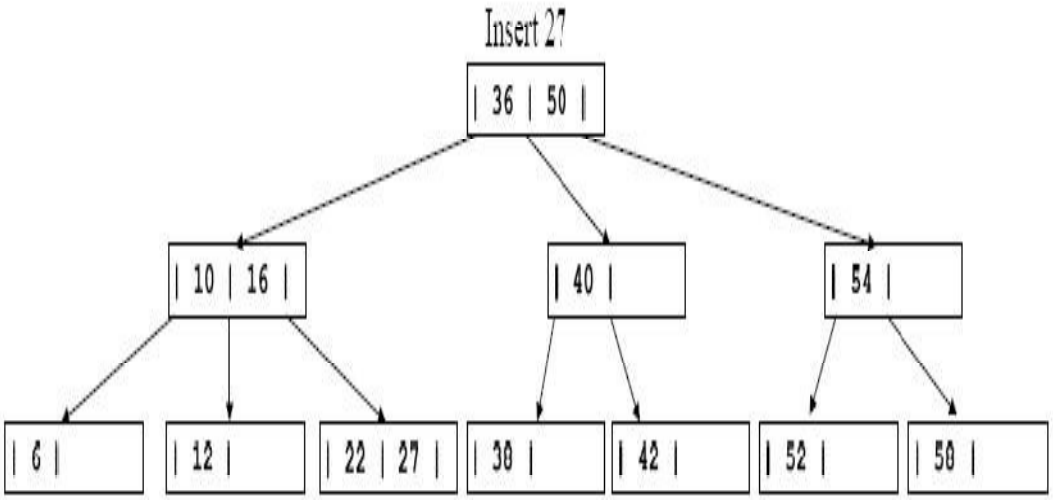
54
10
22
38
40
52
58


Insert 42
36
50
16
40
54
10
22
38
42
52
58


Insert 12
36
50
16
40
54
10
12
22
38
42
52
58


Insert 6
| 36 | 50 |
| 10 | 16 |
| 40 |
| 54 |
| 6 |
| 12 |
| 22 |
| 38 |
| 42 |
| 52 |
| 58 |


Insert 27
| 36 | 50 |
| 10 | 16 |
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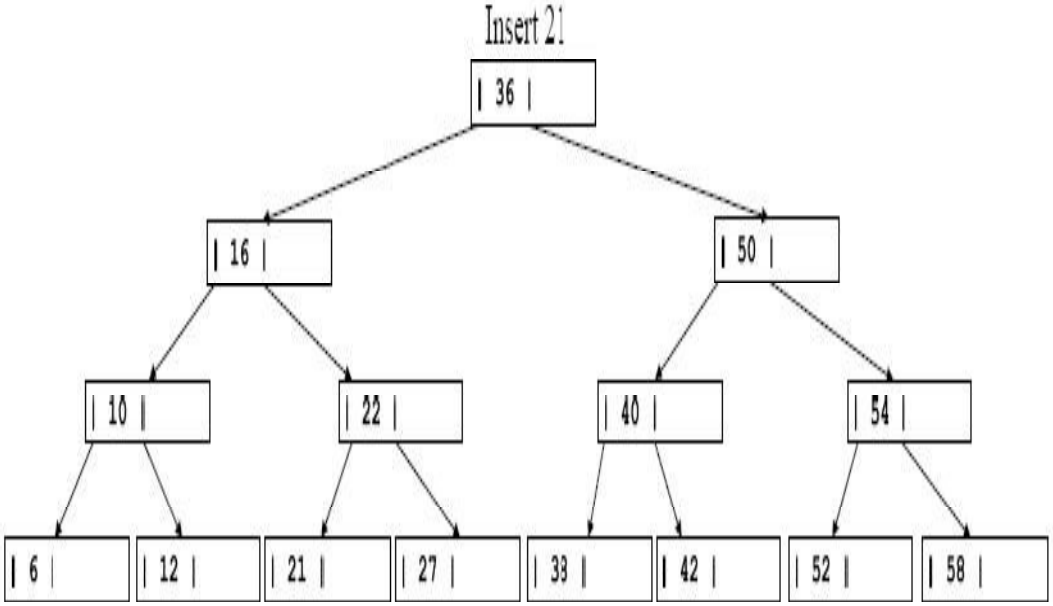
| 42 |
| 52 |
| 58 |


Insert 21
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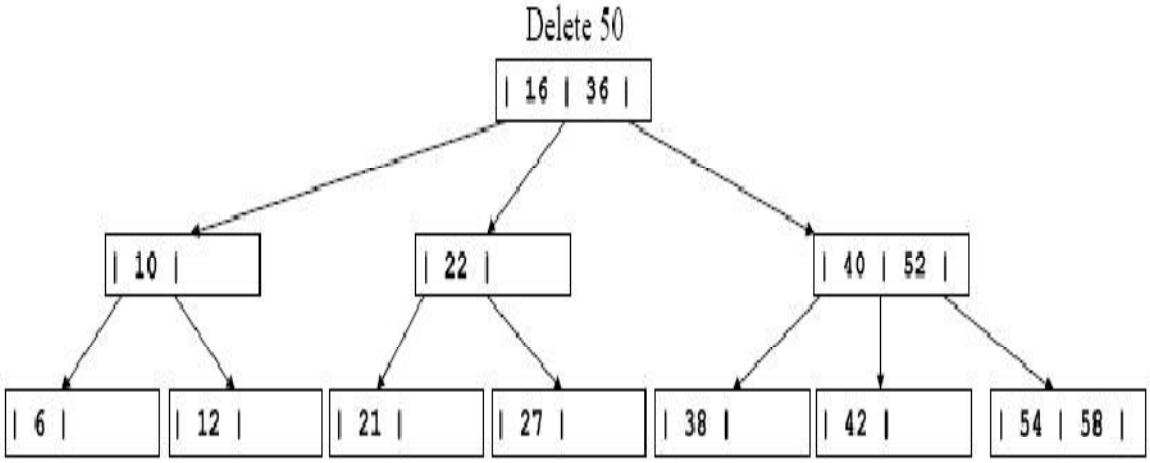
| 42 |
| 52 |
| 58 |


Delete 50
| 16 | 36 |
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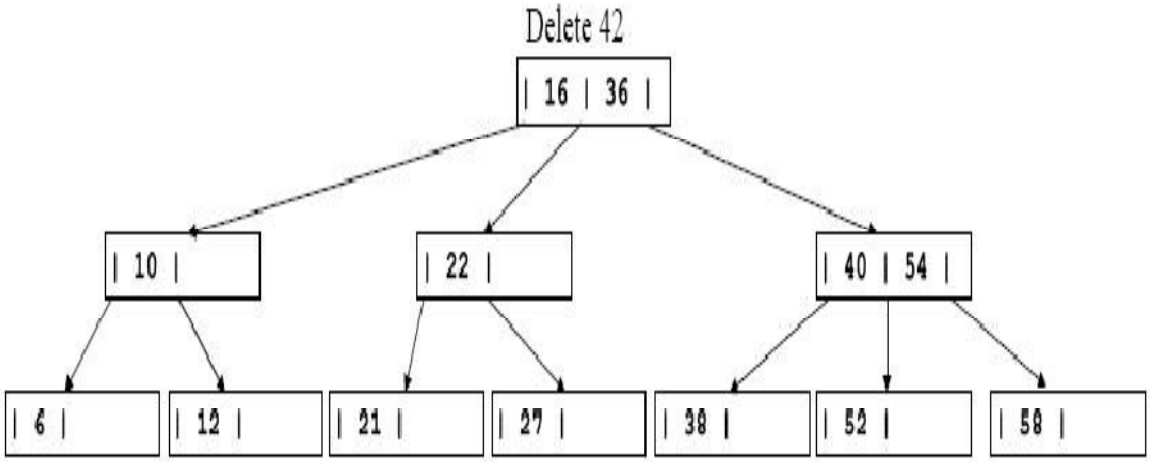
| 54 | 58 |


Delete 42
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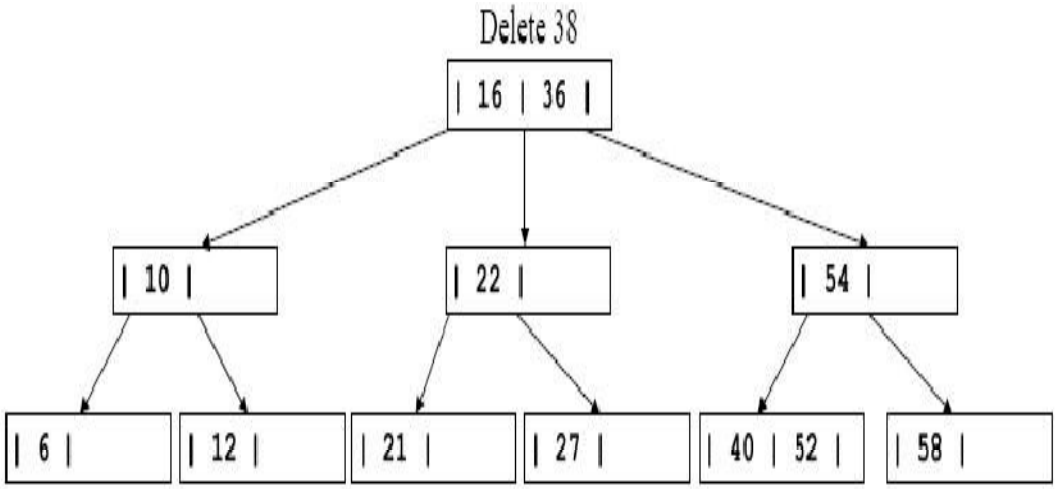
Delete 38
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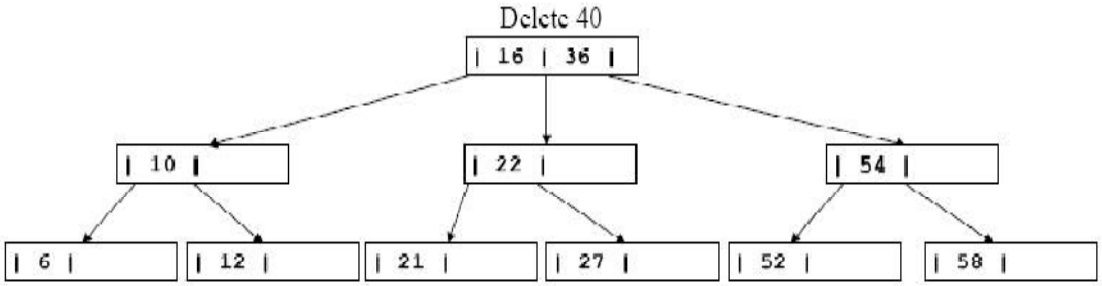
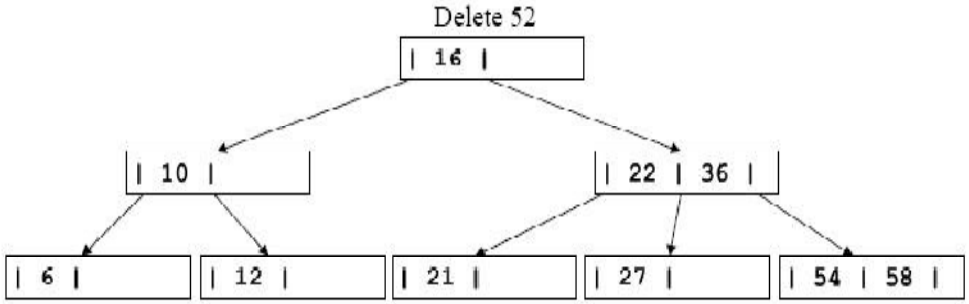
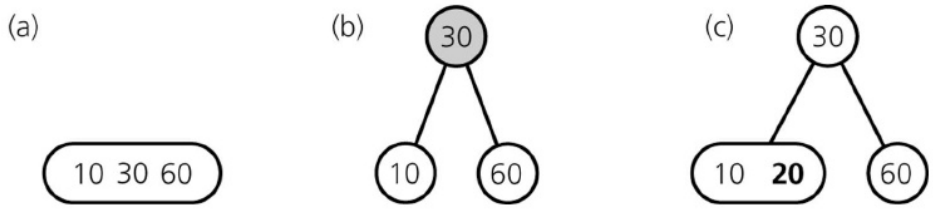
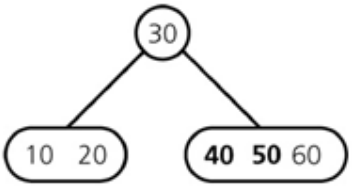




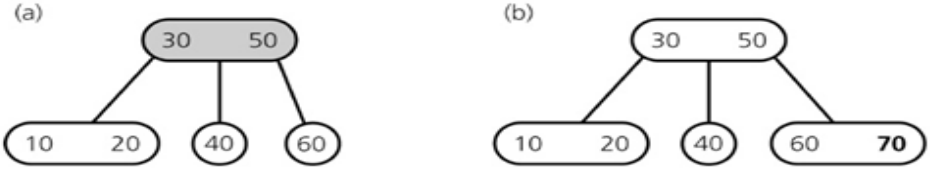
****U B stablo se ubacuju redom 60, 30, 10, 20 ...**



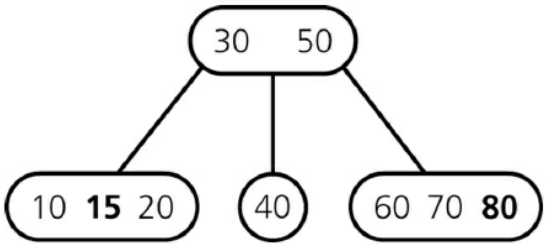
... 50, 40 ...



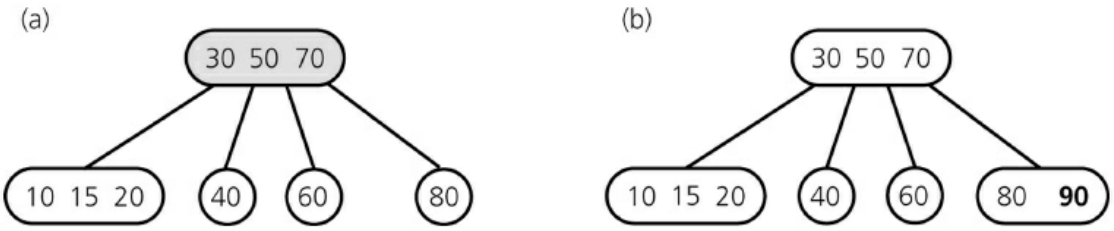
...70...



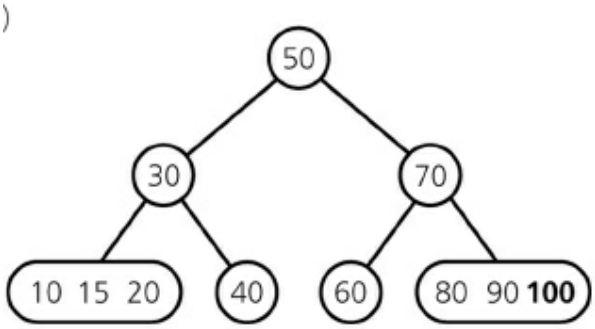
...80 15...



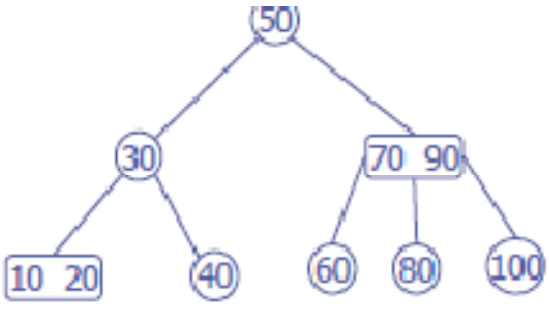
...90...

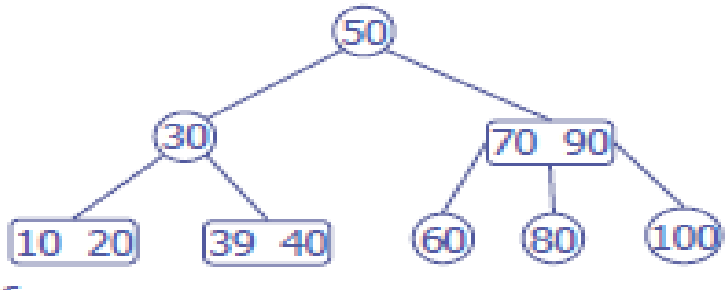


...100...



Neka je dato B stablo u koje se vrsi ubacivanje sledecih elemenata 39 38 36 35 34 33




50
30
70 90
10 20
39 40
60
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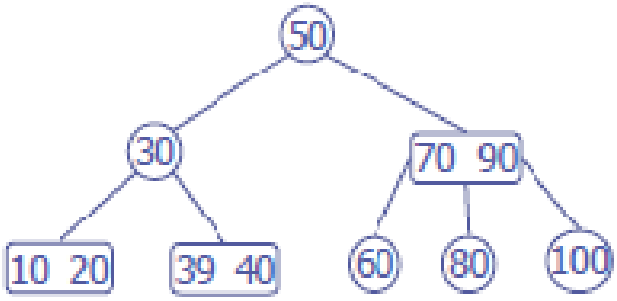
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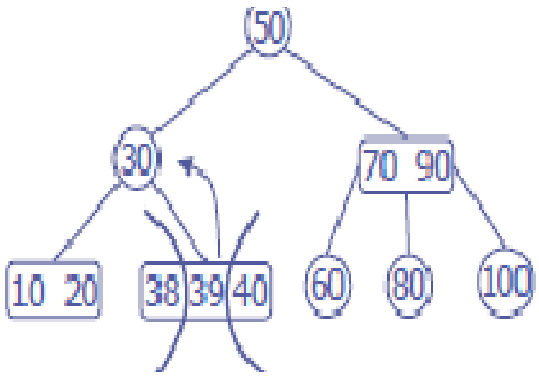
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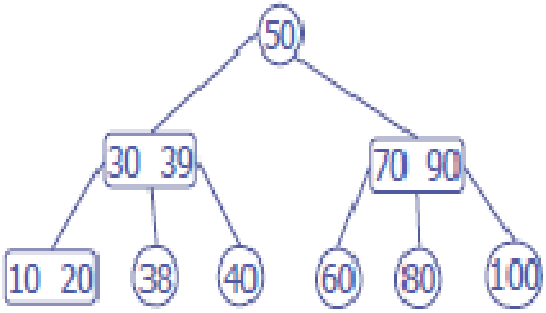
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10 20
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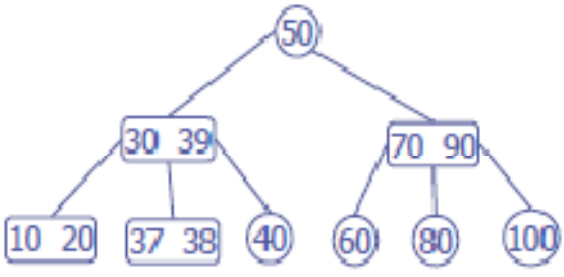
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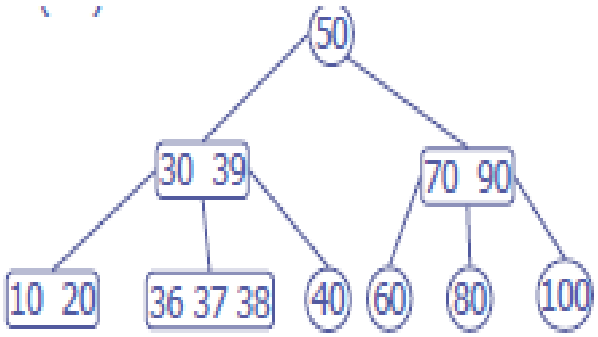
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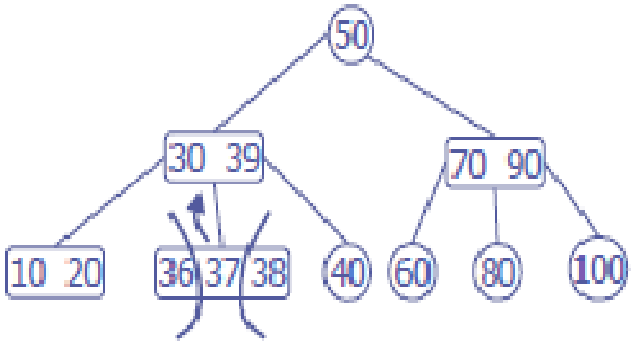
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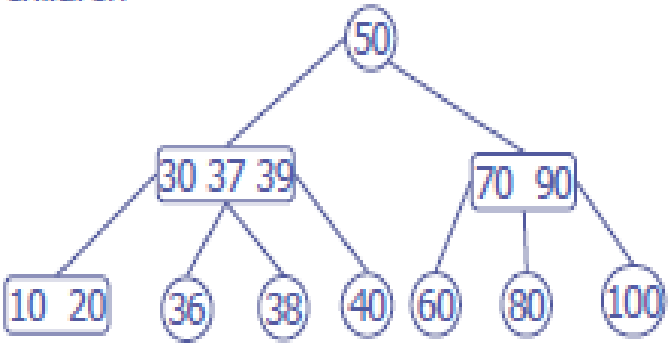
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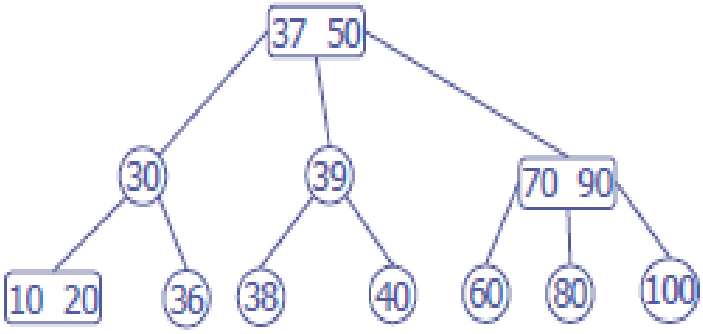
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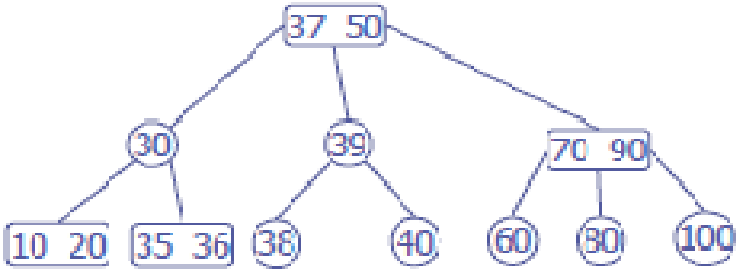
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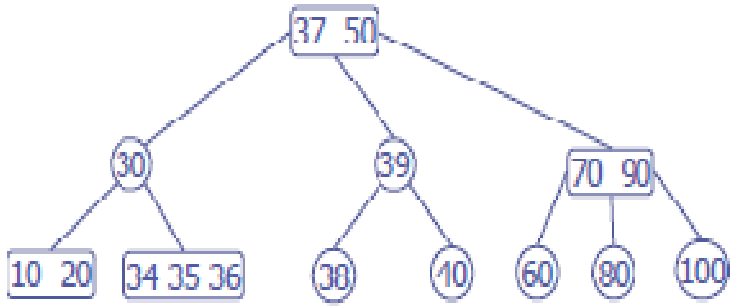
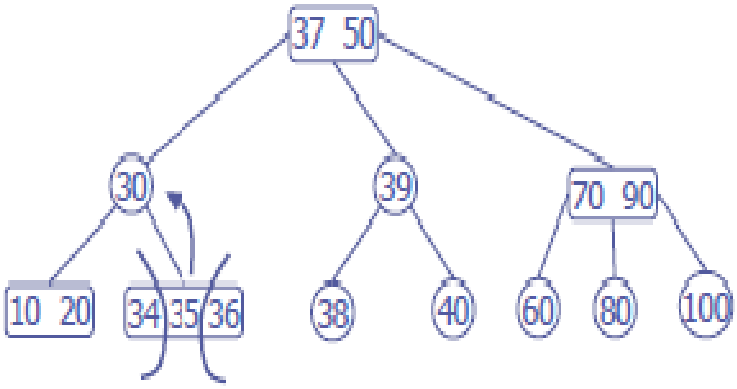
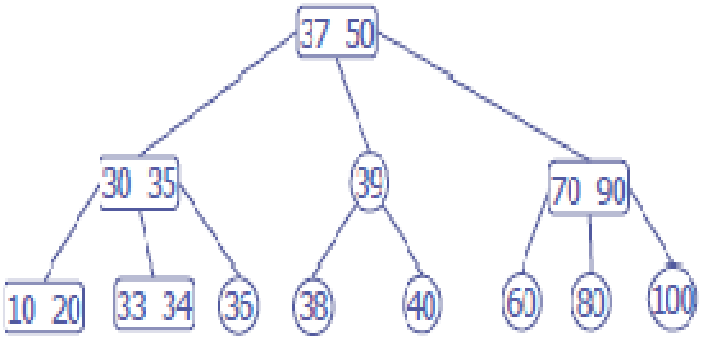
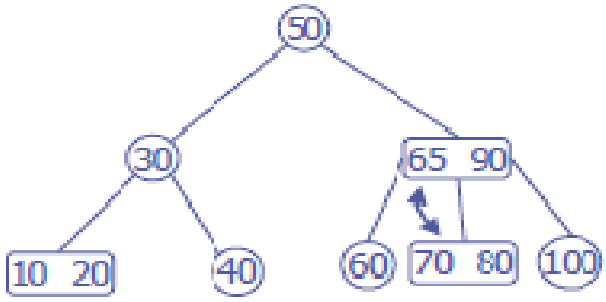
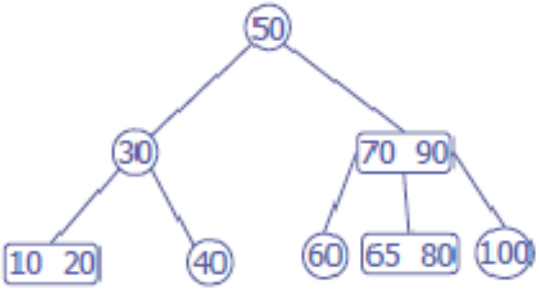


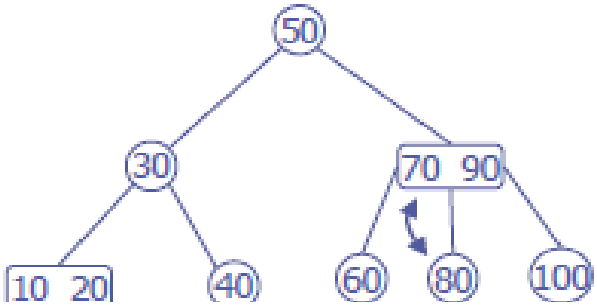




*****Sada se vrsi brisanje sledecih cvorova 65 70 100 80*****



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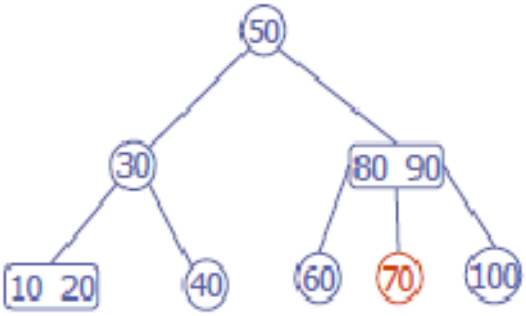
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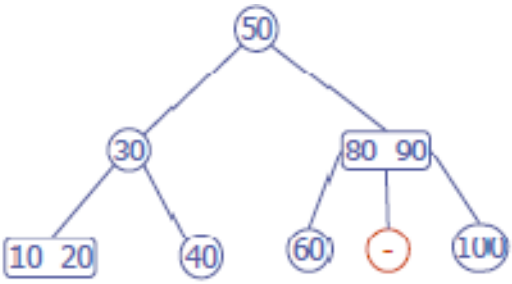
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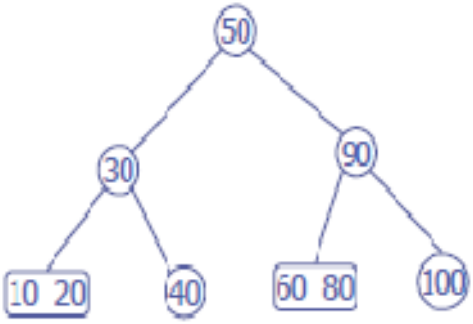
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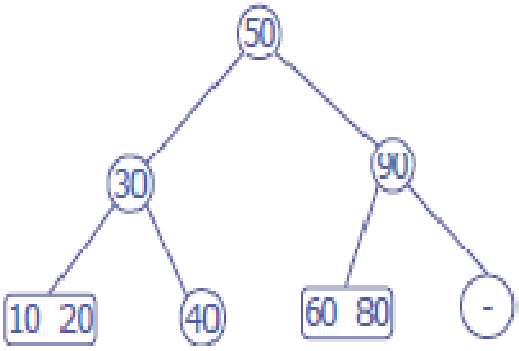
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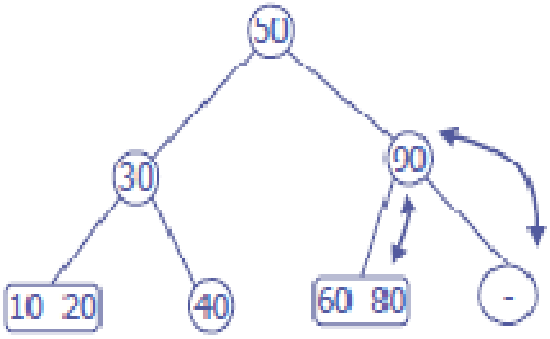
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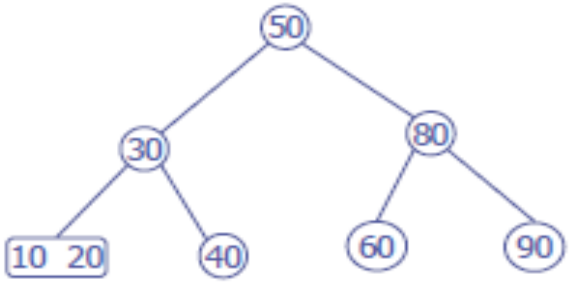
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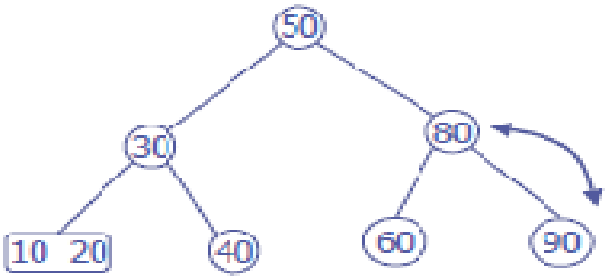
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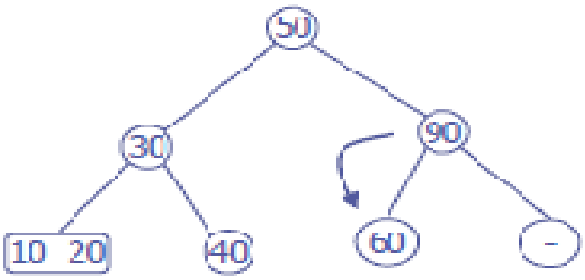
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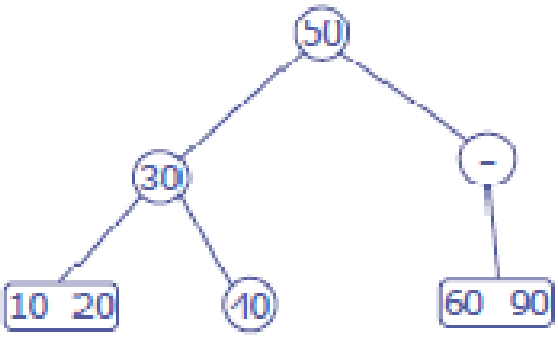
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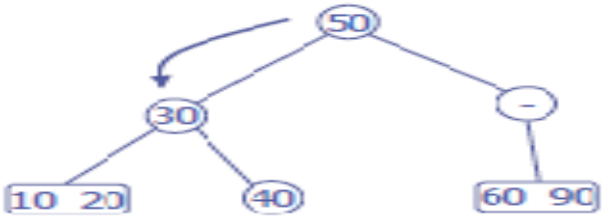
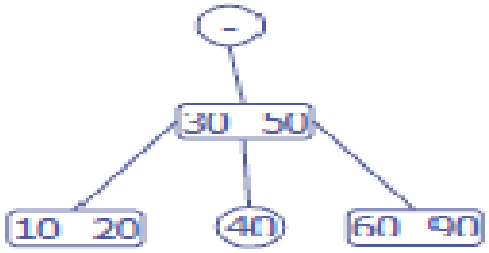
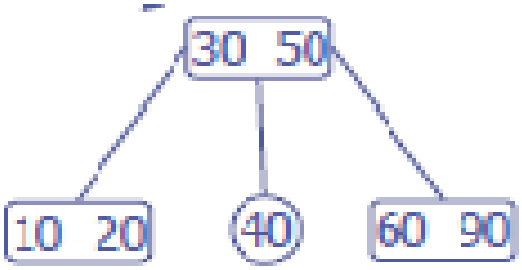






B* stablo

Unošenje (uvek u list) – analogno unošenju u B-stablo, OSIM što:

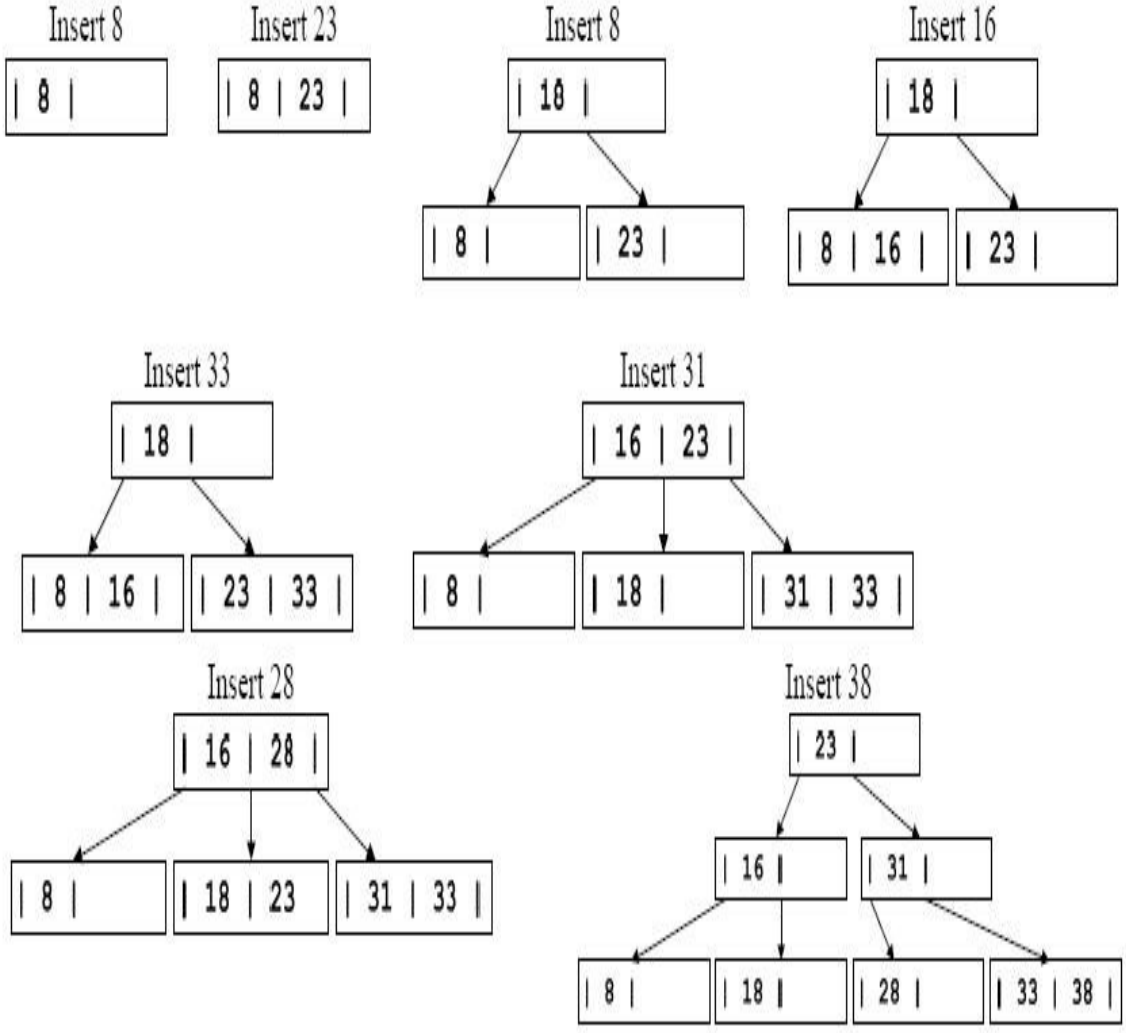
– Ako je list pun, pokušava se *prelivanje* desnim ili levim bratom:
• urede se svi kljucevi iz lista, izabranog brata, razdvojni kljuc (iz neposrednog prethodnika) i kljuc koji se unosi (ukupno m-1+k+1+1 kljuceva)
• floor((m+k+1)/2) kljuceva (manjih ili vecih, u zavisnosti od toga da li se preliva iz desnog ili levog brata) ostaje na starom listu, a ostali idu u desni / levi brat (središnji se “penje”)
– Ako prelivanje ne uspe, cepaju se 2 cvora (i puni list i njegov brat) na 3 cvora, tako što u prvi ide floor((2m-2)/3), u drugi floor((2m-1)/3) a u treci floor((2m)/3) kljuceva, a u cvor neposredni prethodnik idu dva razdvojna kljuca

– Prethodno pravilo se propagira po potrebi sve do korena

**U prazno B*-stablo reda 3 umeću se redom ključevi 8, 23, 18, 16, 33, 31, 28, 38, 48, 40, 57 a zatim se redom brišu ključevi 18, 31, 57, 40. Nacrtati izgled stabla nakon svake od navedenih izmena.

m=3

- svaki čvor osim listova i korena ima najmanje $\lceil \frac{2m-1}{3} \rceil = \lceil \frac{6-1}{3} \rceil = 2$ podstabla
- koren ima najmanje 2, a najviše $2\lfloor \frac{2m-2}{3} \rfloor + 1 = 2\lfloor \frac{6-2}{3} \rfloor + 1 = 3$ podstabla
- nakon podele nekog čvora, ključevi se raspoređuju u odnosu 1-1-2



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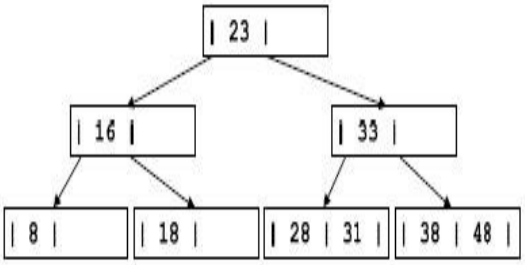

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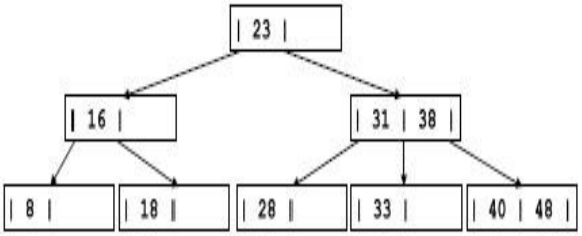

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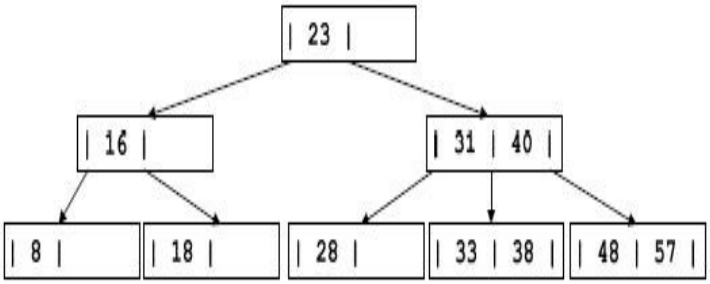

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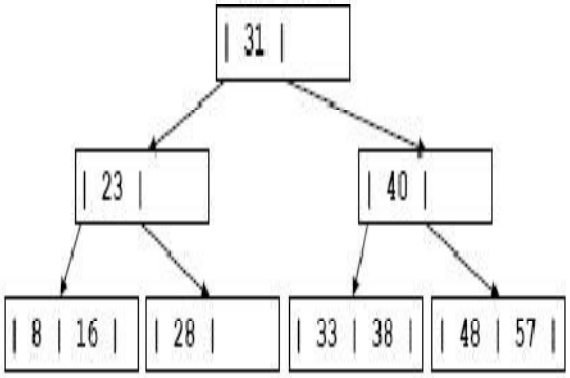

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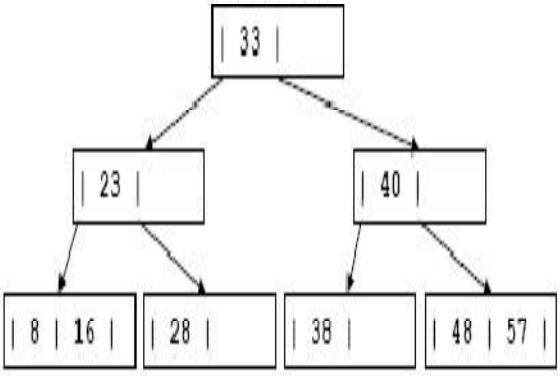

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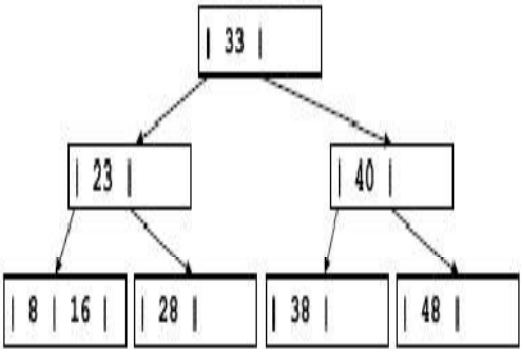

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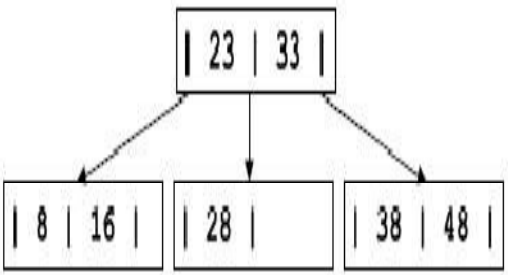
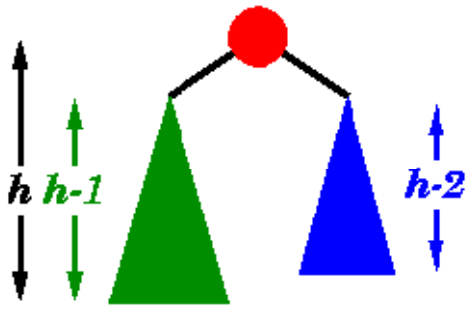


**AVL stablo*

AVL stablo je binarno stablo pretrage kod koga je za svaki čvor apsolutna vrednost razlike visina levog i desnog podstabla manja ili jednaka od jedan.

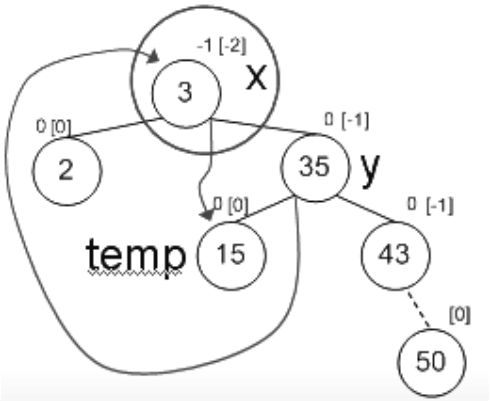
Visina stabla je visina njegovog korena, tj. maksimalno rastojanje od korena do nekog čvora..

Visina čvora *x* je rastojanje od *x* do najdaljeg potomka.



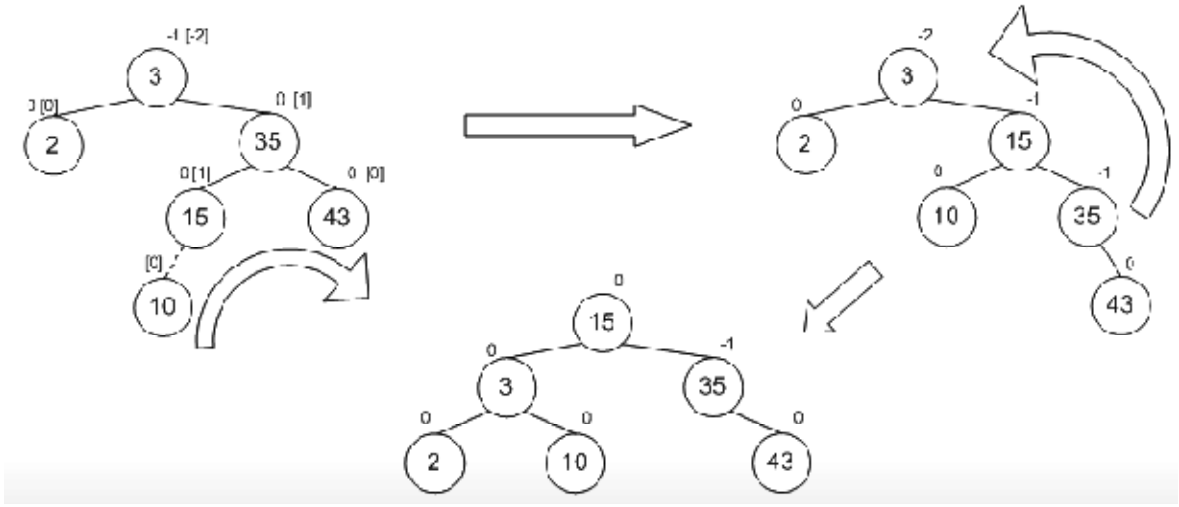
Potreba za rebalansiranjem AVL stabla se javlja kod umetanja lista, kada je neki njegov predak već nagnut (**kritičan čvor**) i nakon umetanja nagnuće prelazi dozvoljenu granicu. Primer rebalansiranja umetanjem čvora 50 na slici u primeru 1:

Nakon umetanja lista, sin kritičnog čvora naginje na istu stranu kao i kritični čvor (potrebna rotacija ulevo ili udesno).

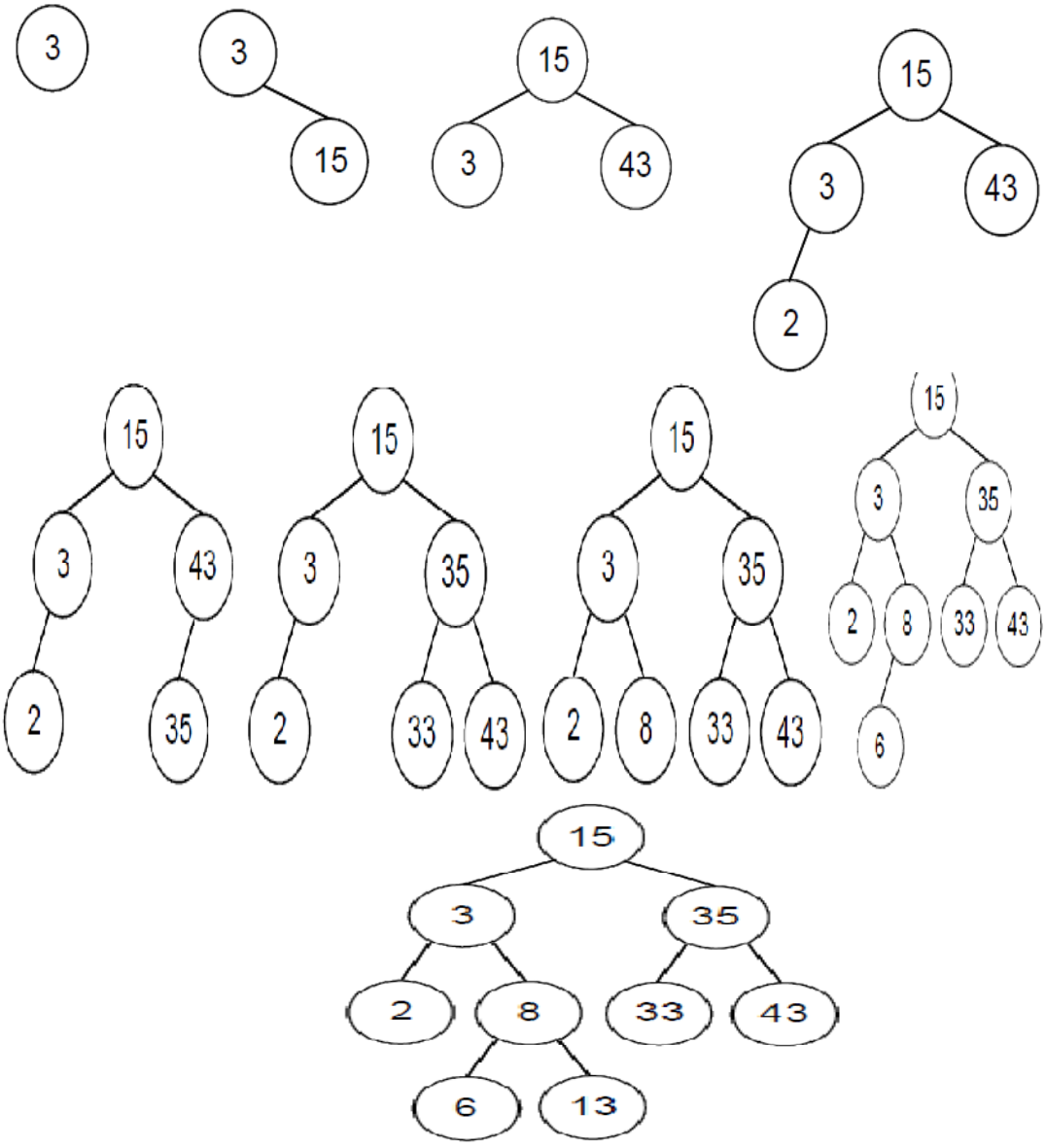


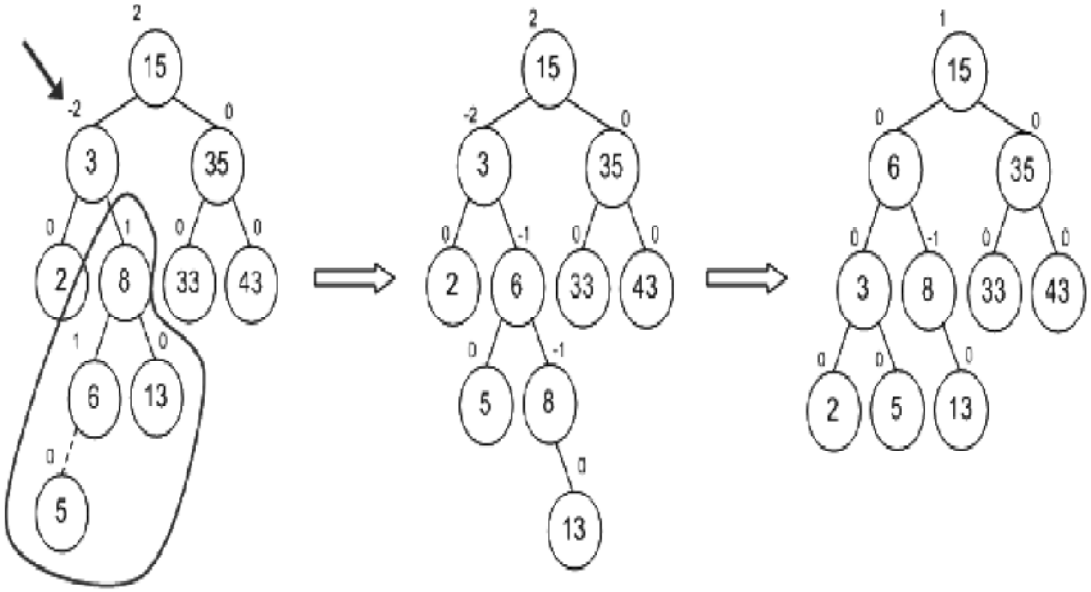
Primer rebalansiranja umetanjem čvora 10 na slici u primeru 1:

Nakon umetanja lista, sin kritičnog čvora naginje na suprotnu stranu od kritičnog čvora (potrebna dvostruka rotacija, u različitim smerovima).



***** Formirati AVl stablo od sledecih brojeva 3 15 43 2 35 33 8 16 13**

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***** Hashing clasa ****

```
public class Hashing {

public int[] niz;
public int brEl;

public Hashing(int dimenzija) {
	niz = new int[dimenzija];
	brEl = 0;
}

public int hashFunkcija(int podatak) {
	return podatak % niz.length; }
```

***** Metoda vrsi ubacivanje novog elementa pomocu hash funkcije*****

```
public void ubaci(int podatak) {

	if (brEl == niz.length)
		return;

	int adresa = hashFunkcija(podatak);

	while (niz[adresa] > 0)
		adresa = (adresa + 1) % niz.length;

	niz[adresa] = podatak;
	brEl++;
}
```

***** Metoda pronalazi trazeni podatak preko hash funkcije*****

```
public int pronadji(int podatak) {
	if (niz.length == 0 || brEl == 0)
		return -1;

	int adresa = hashFunkcija(podatak);
	int pom = adresa;
	while ((niz[adresa] != podatak) && (niz[adresa] != 0)) {
		adresa = (adresa + 1) % niz.length;
		if (adresa == pom)
			return -1;
	}

	if (niz[adresa] == podatak)
		return adresa;
	return -1;
}
```

*****Metoda izbacuje novi element koristeci najpre hash fukciju da ga pronadje*****

```
public void izbaci(int podatak) {
	int adresa = pronadji(podatak);

	if (adresa == -1)
		return;
	niz[adresa] = -1;
}
```

```
	public int hashFunkcija(int podatak) {
		return podatak % niz.length;
 }
```

*****Hashing Olancavanjem*****

```
	public class HasningOlancavanjem {

	public CvorJSListe[] niz;

	public HasningOlancavanjem(int dimenzija) {
		niz = new CvorJSListe[dimenzija];
	}
```

***** Metoda ubacuje podatak u hash niz*****

```
	public void ubaci(int podatak) {

		if (niz.length == 0)
			return;

		int adresa = hashFunkcija(podatak);
		if (niz[adresa] == null)
			niz[adresa] = new CvorJSListe(podatak, null);
		else {
			CvorJSListe pom = niz[adresa];
			while (pom.sledeci != null)
				pom = pom.sledeci;

			pom.sledeci = new CvorJSListe(podatak, null);
		}
	}
```

***** Metoda trazi prosledjenu vrednost i vraca index elementa u nizu, a ako ne postoji, vraca -1 *****

```
	public int pronadji(int podatak) {
		if (niz.length == 0)
			return -1;

		int adresa = hashFunkcija(podatak);
		if (niz[adresa] == null)
			return -1;
		CvorJSListe pom = niz[adresa];
		while ((pom != null) && (pom.podatak != podatak))
			pom = pom.sledeci;

		if (pom == null)
			return -1;
		return adresa;
	}
```

*****Metoda trazi prosledjenu vrednost i vraca CvorJSLIste, ako ne postoji, vraca null*****

```
	public CvorJSListe pronadjiCvor(int podatak) {
		if (niz.length == 0)
			return null;

		int adresa = hashFunkcija(podatak);
		if (niz[adresa] == null)
			return null;
		CvorJSListe pom = niz[adresa];
		while ((pom != null) && (pom.podatak != podatak))
			pom = pom.sledeci;

		return pom;
	}
```

***** Metoda izbacuje prosledjeni podatak*****

```
	public void izbaci(int podatak) {
		int adresa = pronadji(podatak);

		if (adresa == -1)
			return;
		CvorJSListe pom = niz[adresa];
		if (pom.podatak == podatak)
			niz[adresa] = pom.sledeci;
		else {
			while (pom.sledeci.podatak != podatak)
				pom = pom.sledeci;
			pom.sledeci = pom.sledeci.sledeci;
		}
	}
```